

**Advanced Jindalee Tracker:
Probabilistic Data Association
Multiple Model Initiation Filter**

S. B. Colegrove

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ABSTRACT

This paper presents the theory and examples of performance for a new algorithm that initiates tracks using a multiple model Probabilistic Data Association (PDA) filter. The analysis is generalised for the case of multiple non-uniform clutter regions within the measurement data that updates the filter. The algorithm starts multiple parallel PDA filters from a single sensor measurement. Each filter is assigned one of a range of possible target model parameters. To reduce the possibility of clutter measurements forming established tracks, the solution includes a model for a visible target. That is, a target that gives sensor measurements that satisfy one of the target models. Other features included in the algorithm are the selection of a fixed number of nearest measurements and the addition of signal amplitude to the target state vector. The inclusion of signal amplitude is one coordinate that is applicable to the non-uniform clutter model developed in this paper.

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Executive Summary

The reason for developing the filter described in this report was to overcome known deficiencies in the Probabilistic Data Association Filter (PDAF) installed in the Jindalee Over-the-Horizon (OTHR) radar up to mid 1998. The most annoying deficiency was the tracker's repetitive tendency to start moving tracks on transponder returns that are not moving. Because most transponders are near airports, the proliferation of these moving tracks often obscured valid target tracks and increased operator workloads to correct the situation. Another deficiency was the excessive number of false tracks, particularly with the night time ionosphere. An outstanding deficiency not addressed here is the universal problem of tracking systems having limited performance with manoeuvring targets. Some improvement is expected in this area from the steps taken to reduce the false track rate.

The solution to these deficiencies involves the formulation of a new filter for initiating tracks. The tracker used before mid 1998 included initiation in its formulation and started multiple tracks from a peak detection that had not been used for updating other tracks. Each of these filters was for one of the possible target ambiguities and the filters were updated independently. Hence a filter could be updated by measurements that did not obey the filter's target ambiguity model. Because this event was not calculated, spurious tracking occurred under some conditions.

The PDAF tracker described in this report resolves the target's ambiguity model, ie stationary or moving with the correct velocity ambiguity, by having a composite track with a filter for each target model. A fixed number of nearest measurements are selected for each filter model and the filter calculates the probability of a measurement coming from other target models. The target models do not change but propagate forward from the initial measurement used to start the filter. The filter recursively estimates the probability of each model.

The inclusion of track initiation into this filter requires the propagation between the target and the radar to be represented by a "Visible" target. That is, a target that gives measurements that satisfy the tracking filter's measurement and detection model. The estimate of a target's visibility requires preassigned probabilities for changing between the visible and invisible states. The filter's estimate of the target's visibility is used as the basis for decisions on track deletion, promotion and display.

To reduce the false track rate, the filter's noise model is extended to include a clutter region index with each measurement. The clutter regions are defined in a modified peak detector that is integrated with an improved data whitener. The clutter regions have predefined non-uniform distributions. For the Jindalee OTHR, the non-uniform distribution is in signal-to-noise and the clutter regions are defined in the azimuth, range and Doppler dimensions.

Examples of the new tracker's performance show significant:

- reduction in track initiation time,
- reduction in false tracks,
- improved tracking of transponders,

which all give a considerable reduction in operator workload.

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Bren Colegrove completed his PhD in Electrical Engineering at the University of Queensland in 1973. Since 1973 he has been employed as a Research Scientist by DSTO. Up to 1975 he worked on Microwave Radar clutter modelling. Since then he has worked on Over-the-Horizon Radar. This work involved the design and implementation of the Jindalee Stage B Radar. The initial emphasis of his work was radar system design based on cost minimization. Subsequent work involved signal processing, displays and automatic tracking systems. From 1979 to 1994 Dr Colegrove was the Australian National Leader of TTCP Technical Panel KTP-2 which concentrates on advancements in automatic tracking systems. Dr Colegrove's present research activities are into advance automatic tracking systems.

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List of Symbols

a	-	The set of volumes $\{a_c\}_{c=1}^C$
a_c	-	Volume of the intersection $O_c \cap \bigcup_{m=1}^M SI_m$
A	-	The set of clutter region volumes $\{A_c\}_{c=1}^C$
A_c	-	Volume of clutter region O_c
$B_m(k)$	-	Vector of offsets in the state vector for the m^{th} target model
c_i	-	Clutter region index for measurement y_i
C	-	Number of clutter regions
$d_{i,m}$	-	Distance between the predicted position for target model m and measurement y_i
D_m	-	The sub-partition of the measurement sample space which has elements containing detections from a visible target with model m
$E\{.\}$	-	Expected value or mean
$F_m(k)$	-	State transition matrix for target model m at time k .
$f_m(y)$	-	The probability density function for measurements from a target which obeys the m^{th} model.
$g_c(.)$	-	Probability density function for clutter measurements in the clutter region c .
$H_m(k)$	-	Sensor measurement matrix for target model m .
i	-	Measurement index.
I	-	The number of measurements preselected to update a tracking filter model. I is the same for all models and is constant.
IM	-	Number of unique measurements selected for all M models.
k	-	Time index except when it is a subscript
$(k j)$	-	Time k given data to time j .
L	-	The list of clutter region indices that intersect $O_c \cap \bigcup_{m=1}^M SI_m$
m	-	Model index.
M	-	The number of different target models. M is a constant.

n	-	The set of the number of measurements $\{n_c\}_{c=1}^C$
n_c	-	The number of measurements within the intersection $O_c \cap \bigcup_{m=1}^M SI_m$
N	-	The set of the number of measurements in the clutter regions $\{N_c\}_{c=1}^C$
N_c	-	Number of measurements in the clutter region O_c
O_c	-	Clutter region
Pc_c	-	Probability that a clutter measurement is within the region $O_c \cap \bigcup_{m=1}^M SI_m$
Pd_m	-	<i>A priori</i> probability of getting a sensor measurement or target detection given that target model m is visible.
$P(k j)$	-	Error covariance of the state estimate $\hat{x}(k j)$.
$P^m(k j)$	-	Kalman filter error covariance matrix of the state estimate given data from target model m .
$P_m(k j)$	-	Error covariance matrix given the target obeys model m .
Pm_m	-	<i>A priori</i> probability of the target being the m^{th} model.
PSI_m	-	The probability that a target measurement from the m^{th} model is within the region SI_m given it is detected.
Ps_m	-	<i>A priori</i> probability that the target measurement is within the surveillance region S given the target obeys the m^{th} model, is visible and detected.
Pv	-	<i>A priori</i> probability of target visibility.
$Q_m(k)$	-	Covariance matrix for the m^{th} target model's process noise.
$R(k)$	-	Sensor measurement noise covariance matrix.
s_i	-	Component of the measurement vector y_i that has a nonuniform pdf
S	-	The surveillance region that contains the sensor data at time k .
S_m	-	The sub-partition of the measurement sample space whose elements contain a target measurement from model m that is within the surveillance region S .

$S_m(k)$	-	Innovation covariance matrix for target model m at time k .
SI_m	-	The multi-dimensional region contained within S and encompassing the I^{th} nearest sensor measurement to target model m .
$u_m(k)$	-	The m^{th} target model's process noise vector.
$v(k)$	-	Sensor measurement noise vector.
V	-	The sub-partition of the measurement sample space which has elements containing measurements when the target is visible.
W_m	-	Weight matrix for the m^{th} model.
$x(k)$	-	Target state at time k .
$\hat{x}(k j)$	-	State estimate of the target at time k given data to time j .
$\hat{x}_{i,m}(k j)$	-	State estimate of the target at time k given that the target satisfies the m^{th} model and the i^{th} measurement is from the target and data to time j .
y_i	-	The i^{th} measurement in the list of IM unique measurements selected by the M models.
$y_{i,m}$	-	The i^{th} nearest sensor measurement to the track representing model m .
$\tilde{y}_{i,m}$	-	The vector between the predicted position for the m^{th} model and the i^{th} measurement.
$Y(k)$	-	The set of IM unique measurements selected by all M models.
$Y_m(k)$	-	The set of I nearest measurements to the track representing model m .
$X_{i,m}$	-	The sub-partition of the measurement sample space where the i^{th} selected measurement is from model m . The index $i = 0$ denotes no measurement from a target with model m when the target is detected and its measurement is within the surveillance region S .
z_i	-	Component of the measurement vector y_i that has a uniform clutter distribution in the clutter region c_i .
$Z(k-1)$	-	Past track data.
$\beta_{i,m}$	-	Probability of event $\Theta_{i,m}$ given all data.
Δ	-	<i>A priori</i> probability of the target remaining visible from $k - 1$ to k
Δ'	-	<i>A priori</i> probability of the target becoming visible from $k - 1$ to k

- $\Theta_{i,m}$ - The event that measurement y_i is from a target which satisfies model m .
- Ω_m - The sub-partition of the measurement sample space containing those elements when the target satisfies model m .

1. Introduction

In a cluttered environment, track initiation is the most critical aspect of target tracking. Such an environment has no unique identification of the source of returns, ie target or non-target. This ambiguity leads to initiation errors that cause failure to initiate target tracks, track divergence and false tracks. These errors tend to increase in number with increasing clutter density.

The term track initiation, in this paper, applies to the operations from the selection of the first possible target measurement to the formation of a track for display. In 1984 the author provided the first solution[6] to track initiation for the Probabilistic Data Association (PDA) filter by introducing the event of an "Observable Target". Here an observable target produces measurements which satisfy the tracking filter's target and detection model. Because observability is also used in control theory to designate a system whose parameters can be estimated from its output, the term "Observable" is now replaced by the term "Visible". That is, a visible target is one which gives measurements commensurate with the tracking filter models. The visible and invisible states of a target are modelled by *a priori* state transition probabilities. The filter gives an estimate of the target's visibility which, when filtered, is tested against a threshold to decide actions such as the display or reporting of tracks.

The solution to initiation in references 6 and 7 also includes starting multiple filters from a single sensor measurement. Each filter has a unique initial velocity within the required velocity range. After a preset period of tracking to resolve the correct velocity option, the filter with the highest confidence (or filtered visibility) above a preset threshold becomes an established track. The filter given in reference 6 was implemented on the Jindalee Over-the-Horizon Radar (OTHR) which includes target Doppler, range and azimuth in its measurements. The Doppler of a measurement selected to start a track is used with the waveform velocity ambiguity to start multiple tracks at each valid ambiguity. A zero radial velocity option for beacon and calibrate signals is also included in this initiation procedure.

Under some conditions this approach initiated tracks at the wrong ambiguity. This occurred more frequently when the difference between low radial velocity targets and beacons could not be reliably resolved in a short time, ie less than 10 track updates. This aspect is illustrated in Figure 1 which shows the range-rate projections from an initial detection. These projections are those derived from two Doppler ambiguities with a calibrate/beacon line included. Also, illustrated in Figure 1 is an approximation of the measurement scatter about each projection. During the early stages of

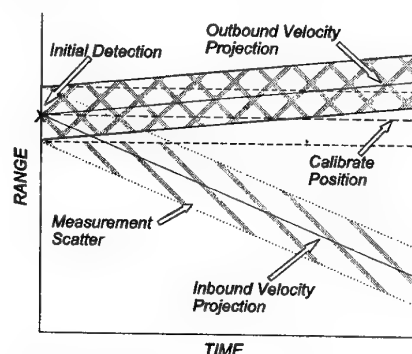


Figure 1. Example of the radial velocity projections from an initial detection.

tracking, a filter can be updated by target measurements which do not satisfy the filter's target model. For example the calibrate track can be updated for a significant time by measurements from an outbound target. As this event is not considered by the filter, erroneous performance results, that leads to the possibility of selecting the wrong ambiguity with subsequent track divergence.

To reduce the incidence of tracks at the wrong ambiguity, the PDA filter approach outlined in references 6 and 7 is extended in this paper to remove the independent update of the multiple filters. The extension involves taking account of the other filter models during the track initiation period.

A number of papers have appeared on multiple model filters and one which includes the PDA technique is developed by Houles and Bar-Shalom in reference 3. Here the target has a fixed number of models with a transition matrix to represent the Markov switching of models between each time step. By also retaining a fixed number of models after each time step, they can stop an exponential growth in the number of model sequences with time. This is achieved by calculating a mixed estimate for each model before the filter update. A further extension by Bar-Shalom and Lerro [5] to include track initiation, introduced another model to represent a "no target". That is, there is a "true" target with one of $M-1$ models or a "no target" with the remaining model. These models are updated by measurements selected from those within a gate placed about the merged estimate of the predicted target position.

Reference 5 also includes signal amplitude in the event probability calculations. The way amplitude is included is taken from the approach by Lerro and Bar-Shalom in reference 4. This approach requires the definition of an expected signal-to-noise ratio (SNR) which is used in the likelihood ratio calculation for a target measurement. When the target has an expected SNR which is different to the assumed value, there is the possibility of sub-optimal performance such as reduced track initiation sensitivity.

The filter developed in this paper for track initiation with a multiple model filter incorporates several previous features with some modification. The previous features included are: visible/invisible target states; tracking with signal amplitude; and, track update based on a nearest neighbour's rule. The modifications do not add a separate model to an invisible target. Rather, the target is either visible or invisible and in either case it is represented by one of M models. Since only one of the models is possible, there is no Markov switching between the models. Also by not assigning a separate target model to the invisible target state, the visible target trajectory is not deviated when its visibility decreases. This is a key distinguishing feature of this new initiation scheme.

Another key feature in this new filter is in how signal amplitude is included. The approach is that used in a classified report by Colegrove et al[9] and published later in open literature[10,11]. It treats signal amplitude like any other tracking co-ordinate such as range or bearing. By including signal amplitude in the target state vector, the filter gives an estimate without the need for an *a priori* assumption about its level[4]. To add signal amplitude, the filter solution must incorporate a non-uniform cluttered environment. This paper uses the method of reference 9 where the solution is given for clutter having an *a priori* arbitrary distribution. Signal amplitude is then just a special case of a general non-uniform clutter model. With this approach amplitude is

incorporated into the selection process in that it is part of the distance calculation between the sensor measurements and the filter predicted position. This is particularly important when selecting the I nearest measurements for filter update. For a high signal-to-noise ratio target, this stops nearby low amplitude clutter measurements being selected for filter update.

The following development of the multiple model track initiation filter starts by outlining the approach in Section 2. Section 3 defines the target, environmental and sensor models on which the filter is based. This is an important aspect of the filter formulation as it lays the ground for the definition of the target and sensor model. From these definitions the equivalent sample space representation of the sensor and environment model is given in section 4. From these preliminaries, the solution for the new multiple model tracking filter is given in section 5. Section 6 then uses the filter solution to show the steps involved in track initiation.

2. Track Initiation Approach

The purpose in using a multiple model filter for track initiation is to limit the initial size of the error covariances when starting a filter with a single possible target measurement. The errors are reduced by subdividing the parameter range for the state vector components which are either not measured by the sensor or ambiguous. A separate filter is then initiated using the parameters for each subdivision with other relevant data. For example, take the case of a sensor required to track targets between v_{\min} and v_{\max} . If range-rate is not measured, the approach involves dividing the radial velocity range into say four equal velocity steps with the covariance based on the step size. The multiple model filter then has four filters where each starts at the position of the initiating sensor measurement and has an initial velocity equal to one of the four subdivisions. For a sensor which measures Doppler, the velocity of each filter is obtained from the unambiguous velocity of the initiating measurement with the appropriate multiple of the sensor ambiguity velocity added or subtracted as illustrated in Figure 1. The parallel filters started from the single measurement are propagated forward until the tracking algorithm can determine which parameter set is valid. Here each parallel filter corresponds to a target model of which there are a fixed number of M models.

At track initiation, a track is defined as tentative and after a predefined time the track's visibility is tested. When the visibility is greater than the valid target threshold, the track enters the pending state. In this state the multiple model filter operations continue and the pending track is presented for display and operator action. When the pending track has a visibility greater than the valid track threshold and when a model's probability dominates, the track is transferred to the established state. Here an alternative tracking filter is employed using the parameters of the dominant model.

These track states are illustrated in the state transition diagram in Figure 2 with the addition of a Dead state. A track becomes dead when its confidence falls below a preset threshold. This test is represented by the transition T4. No recovery is possible from this state which can be entered from any one of the three active states. The transition T1 denotes the multiple model filter update step. The transitions T2 and T3 denote the rules for track transfer from the Tentative and Pending states.

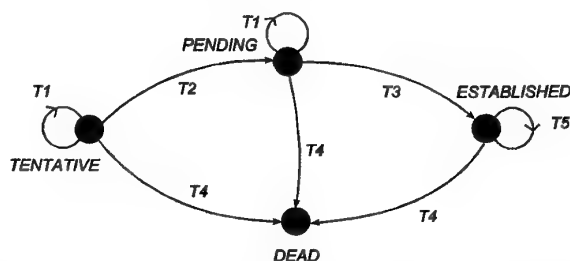


Figure 2. State transition diagram for a track after initiation

The filter developed in the following sections is applicable to both the Tentative and Pending states T1 in Figure 2. The filter for the Established state T5 is not considered here.

3. Model of Environment and Sensor

The solution to the multiple model track initiation filter starts with an environment and sensor model. The sensor model contains the definition of the track data and the method of selecting the sensor measurements for updating the tracking filters. Included with these definitions are probability terms which are required later for the filter formulation.

Figure 3 gives a diagrammatic representation of the environment and sensor. The environment contains a range of targets, various clutter objects and noise sources. These may be illuminated by the sensor or they may generate their own characteristic emissions. The sensor collects returns from the environment via the propagation medium and converts them into sensor measurements. Each measurement is a vector which contains a list of parameters such as range and bearing. The aim here is to give only measurements which have a high probability of being from targets. In many practical applications this is difficult to achieve, particularly when clutter objects have target-like returns.

Here the assumption is made that the sensor measurements are dominated by clutter measurements. The measurement process also supplies and updates the clutter model parameters that are used later in the track update process.

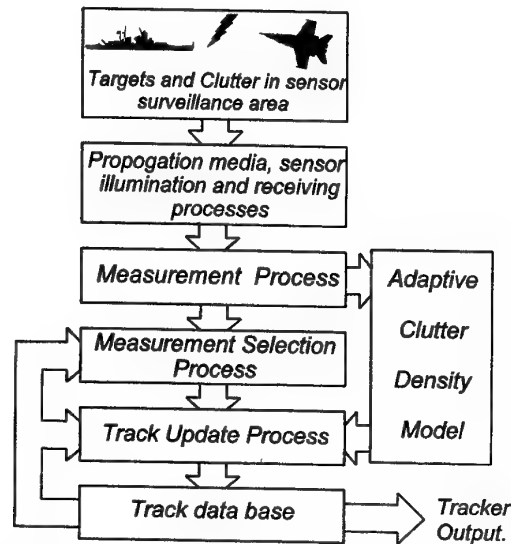


Figure 3. Sensor and Environment Model.

The selection of these sensor measurements for updating the tracking filter is done by the measurement selection process. It uses the predicted track positions from tracks stored in the track data base. The final step is track update which uses the track data, the selected measurements and the clutter model parameters.

3.1. Track Data

The track data base contains *a priori* data for the state estimate and probability of each target model. Tracks are assumed to be independent of one another. A track has a fixed number of M state estimates, associated covariances and model probabilities. These are assumed to be a sufficient statistic to summarise all previous target data derived from the sensor's measurements. For each track, all past data up to time $k-1$ are denoted by the symbol $Z(k-1) = \{Z_m(k-1)\}_{m=1}^M$.

The target model's state estimate for time ' k ' based on past data up to time ' j ', is denoted by $\hat{x}_m(k|j)$ and the covariance by $P_m(k|j)$ where m is the model index number. The collection of M estimates are called a track and each target in the surveillance region has an associated track representation. A track's estimate of a target is denoted by $\hat{x}(k|j)$ and the covariance by $P(k|j)$.

The symbol for the *a priori* probability of the target being represented by model m is Pm_m . The model probabilities are constrained by $\sum_{m=1}^M Pm_m = 1$. For the proposed initiation scheme, there is no transition between models. Therefore $Pm_m(k|k-1) = Pm_m(k-1|k-1)$. When no *a priori* model data are available, the initial model probability is given by $Pm_m(0|0) = \frac{1}{M}$.

Sometimes, the actual target model may lie between two nearby models in the list of M . This leads to the nearby models having similar state estimates with shared model probabilities. The duplicate model with model probability Pm_m must be removed by setting its probability to zero. The probability of the duplicate model is then added to the remaining $M - 1$ models by scaling up their probabilities by $1/(1 - Pm_m)$.

3.2. Target Model

The equations of motion give the time dependence of the target's state with perturbations added to account for target dynamics. The M target models form a mutually exclusive and exhaustive set of possible models for describing the dynamics of the target. The m^{th} target model holds throughout the tentative and pending track period and is given by,

$$x_m(k+1) = F_m(k)x_m(k) + B_m(k) + u_m(k), \quad m=1, \dots, M. \quad (1)$$

Where

- $x_m(j)$ is the state of a target with model m at time j .
- $F_m(k)$ is the state transition matrix for the m^{th} target model.
- $B_m(k)$ is a vector containing known offsets that are applied to the state vector for the m^{th} target model. This vector contains the parameters of the model subdivision (see section 2).
- $u_m(k)$ is the m^{th} target model's process noise vector, Gaussian distributed with zero mean and covariance $Q_m(k)$. The process noise accounts for target manoeuvres and model errors.

Here $F_m(k)$, $B_m(k)$ and $Q_m(k)$ are assumed to be known for each model.

The multiple model approach by Bar-Shalom et al[5] has a "no target" model added to the model list with model switching probabilities represented by a Markov transition matrix. This contrasts with the approach in this paper where the "Visible" and "Invisible" target model in [7] is part of the process that represents the Environment and Propagation Process in the next section.

3.3. Environment and Propagation Process

No matter which of the M models is correct, the target is either visible or not visible to the sensor. The definition for target "Visibility" is given by the following.

A "Visible" target gives sensor measurements that satisfy one of the sensor target and measurement models.

An "Invisible" target gives no sensor measurements that satisfy any of the sensor target models for $m=1, \dots, M$.

The definition for an invisible target includes the case of a target that gives measurements but these do not satisfy the sensor models. That is, its measurements are assumed to be equivalent to those from clutter.

The factors that determine the transition between the visible and invisible states of a target are independent of the model. Following the approach in reference 6, the *a priori* probability that the target is visible is given by,

$$\begin{aligned} P_v &= P_v(k-1|k-1)\Delta(k) + (1 - P_v(k-1|k-1))\Delta'(k) \\ &= (\Delta(k) - \Delta'(k))P_v(k-1|k-1) + \Delta'(k). \end{aligned} \quad (2)$$

Where $P_v(k|j)$ is the probability of target visibility.

$\Delta(k)$ is the probability of the target remaining visible from time $k-1$ to k .

$\Delta'(k)$ is the probability of the target changing from being invisible at time $k-1$ to being visible at time k .

For notational simplicity P_v represents the *a priori* probability $P_v(k|k-1)$. When denoting the *a posteriori* probability, the time index is included, ie $P_v(k|k)$.

The state transition probabilities $\Delta(k)$ and $\Delta'(k)$ are design parameters which are derived by experiment. There is no requirement to keep them fixed for all tracks and time. For example, the probability of a target remaining visible is higher after sustained tracking than it is during track initiation. The reason for a low $\Delta(k)$ is to cater for the case of a track started from a clutter measurement. This is equivalent to a target being visible for the initiating measurement and invisible thereafter.

3.4. Measurement Process

The measurement process for a microwave radar is the plot extractor while for an OTHR it is the peak detection and interpolation step. The data input to the measurement process is the received and processed signal in each resolution cell. The signal is the sum of the returns from targets, clutter and noise. Clutter and noise are collectively called clutter. The assumed functions of the measurement process are as follows.

- (1) Convert the receiving system output into measurements that contain estimates of parameters such as range and bearing.
- (2) Partition the receiving system output into clutter regions in which measurements from clutter are represented by a common probability density function (pdf). Determine the clutter region parameters and update the data for the clutter region pdf models.

The measurements output from step (1) are represented by the set $\{y_i\}_{i=1}^N$, where y_i is the i^{th} measurement and N is the number of measurements at time k . Here the time index is omitted for clarity. For the Jindalee tracker, these measurements are supplied by an enhanced peak detector. Its description and performance is given in [12].

Most formulations for the PDA filter use a uniform pdf for clutter [1 . . 5]. This assumption is not always valid [11]. Step (2) is a new innovation that caters for multiple nonuniform clutter regions. Details of the various actions to identify the clutter regions and determine their parameters are found in [14].

The next section describes the measurement process parameters required by the tracking filter for target and clutter measurements.

3.4.1. Target Probabilities

When a target is visible, both target signal fluctuations and the conditional tests in the measurement process can stop the formation of a target measurement. The action of getting a target measurement is called detection and is represented by the *a priori* probability Pd_m where,

Pd_m is the Probability of obtaining a sensor measurement when a target is visible and obeys the m^{th} model.

The value chosen for Pd_m is based on those data variables which are not in the measurement vector y (see section 3.5.1). This is best illustrated with the example of tracking with signal level included as one of the estimated parameters. Here signal level is treated the same as range or azimuth in the selection process. Therefore the detection threshold is one of the limits for the selection probability calculation of Ps_m in section 3.5. The value for Pd_m is then based on factors such as the measurement process not being overloaded and this probability is close to unity. Normally Pd_m is the same for all values of m .

The sensor measurements from Visible targets are related to the target state by,

$$y_m(k) = \mathbf{H}_m(k)x_m(k) + v(k). \quad (3)$$

Where $y(k)$ is the measurement vector,

$\mathbf{H}_m(k)$ is the measurement matrix for model m ,

$v(k)$ is the measurement noise vector, Gaussian distributed with zero mean and covariance $R(k)$.

Here $H_m(k)$ and $R(k)$ are assumed known at time k . The model index for H is included to allow for cases such as fixed transponder signals in Doppler radars. Here the measured Doppler bears no relation to target range-rate.

The pdf for target measurements is defined given the target obeys the m^{th} model. It is expressed with respect to the predicted position of the track. The distance of a measurement from this position is normalised by the innovation covariance for the m^{th} target model $S_m(k)$, where $S_m(k) = H_m(k)P_m(k|k-1)H_m^T(k) + R(k)$. The equation for calculating this distance is,

$$d_{i,m}^2 = \tilde{y}_{i,m}^T S_m^{-1} \tilde{y}_{i,m} \quad (4)$$

Where $\tilde{y}_{i,m}$ is the vector between the track predicted position for model m , ie $\hat{x}_m(k|k-1)$, and the i^{th} measurement, ie $\tilde{y}_{i,m} = y_i - H_m \hat{x}_m(k|k-1)$.

The pdf of target measurements is then given by the following expression,

$$f_m(y_i) = \frac{e^{(-d_{i,m}^2/2)}}{(2\pi)^{\mu/2} \sqrt{|S_m|}} \quad (5)$$

Where μ is number of degrees of freedom, ie the number of components in the measurement vector.

This distribution is not truncated and therefore has the following property.

$$\int_{\forall y} f_m(y) dy = 1 \quad (6)$$

3.4.2. Clutter Regions and Probabilities

The first step towards getting the clutter probability density is to determine the coordinates in which clutter measurements have a uniform distribution. The remaining measurement coordinates then have a nonuniform distribution for clutter measurements. For the measurement coordinates with a uniform distribution, the surveillance region is partitioned into regions that can be modelled by the same pdf in the nonuniform coordinates[13]. Two clutter regions are illustrated in Figure 4 where the clutter in each

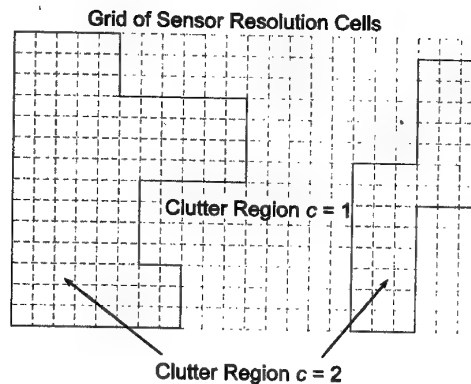


Figure 4. Example of two clutter regions.

region is uniformly distributed. C denotes the total number of clutter regions and the symbol for a clutter region is O_c where $c = 1, \dots, C$. For each measurement y_i there is an associated clutter region index c_i . The number of measurements contained in the clutter region O_c is N_c and its volume is given by A_c . The sets representing all clutter regions, measurements and volumes are respectively $O = \{O_c\}_{c=1}^C$, $N = \{N_c\}_{c=1}^C$ and $A = \{A_c\}_{c=1}^C$.

The main measurement coordinate with a nonuniform distribution is signal-to-noise ratio (SNR). The coordinates that have a nonuniform distribution are represented by s_i so that the measurement vector is given by $y_i = \{z_i, s_i\}$. Here z_i contains the components that have a uniform clutter distribution in the clutter region with index c_i .

Clutter measurements produced by the sensor are assumed to be independent of one another. The *a priori* probability distribution of a clutter measurement in the s variables given it is in clutter region O_c is given by the function $g_c(s)$. In the z coordinates the distribution is uniform so that the pdf of clutter measurements is given by

$$g_c(y) = \frac{g_c(s)}{A_c}.$$

3.4.3. Clutter Model Parameters

The parameters retained for use in the track update process are the clutter regions, number of measurements in each and their volumes. The clutter regions are best represented by an array, similar to that output by the receiving process except with only the coordinates of z . The elements of this array contain the clutter region index c . This array is used later for the calculating the intersection of the clutter region with the region that contains the selected measurements.

In addition to the clutter region data, filtered histograms are retained for each clutter region. A simple low pass filter for each clutter region c with weight $w_c(k)$ is applied to the raw histogram data from the measurements $\{y_i\}_{i=1}^N$. The estimated histogram for bin b is given by [14]:

$$\hat{h}_c^b(k) = \hat{h}_c^b(k-1) + w_c(k) [h_c^b(k) - \hat{h}_c^b(k-1)] \quad (7)$$

where $h_c^b(k)$ is the number of measurements at time k for bin b and clutter region c . This histogram data is used to compute the pdf $g_c(s)$ as described in [14]. This density is considered *a priori* because it is derived from filtered past data and because the number of measurements used for its estimate are much greater than the number selected by a track.

3.5. Measurement Selection Process

The selection process is based on the nearest neighbours rule[6,7] where a fixed number of measurements are always selected at each time step. During initiation the valid model is unknown and the two alternative ways for selecting the nearest measurements are:

- (a) From the measurement error \mathbf{R} , predicted position $\hat{\mathbf{x}}(k|k-1)$ and covariance $\mathbf{P}(k|k-1)$ of the target (see Section 3.1), select the ' I ' nearest measurements.
- (b) From the measurement error, predicted position $\hat{\mathbf{x}}_m(k|k-1)$ and covariance $\mathbf{P}_m(k|k-1)$ of each model, select the I nearest measurements to each of the M models.

In the first approach, the covariance $\mathbf{P}(k|k-1)$ is for a weighted sum of Gaussians. The selection step then uses a single Gaussian pdf to choose measurements closest to the expected target position. For modest values of I , models with a low probability will not have nearby measurements chosen. This problem is avoided by the second approach which uses the M Gaussian pdf's retained by the filter.

Approach (b) always chooses the statistically significant measurements to each model and avoids any problems associated with approximating a Gaussian mixture by a single Gaussian. However when the measurement scatter from the models overlap, as illustrated in Figure 1, it is likely that there will be measurements common to more than one model. This is most likely immediately after the start of a track.

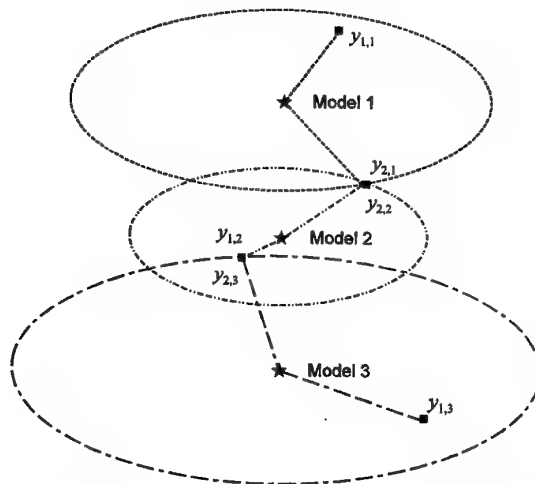


Figure 5. Example of selection process for $I=2$ and $M=3$

Figure 5 gives an example of measurements being shared between the models for the case of $I = 2$ and $M = 3$. Here the measurements y_i that are selected have a double subscript i,m that denotes the order of the measurement and the model number. To illustrate the shared measurements with this notation, the above example has $y_{2,1}$ as the second nearest measurement to model 1 and it is the same as $y_{2,2}$ which is the second nearest to model 2. A similar case occurs where $y_{1,2}$ and $y_{2,3}$ are common. As each

model is updated by all measurements, the duplicate measurements are removed to form a new list of measurements. This list is represented with a non-italicized y , namely $y_i, i = 1, \dots, IM$ where IM is the number of unique measurements. In the example in Figure 5, the first two y values are: $y_1 = y_{1,1}, y_2 = y_{2,1}$ and $y_{2,2}$ is not included in the list.

The region that contains the I nearest measurements to model m is represented by¹ SI_m and the set of all such regions by $SI = \{SI_m\}_{m=1}^M$. The analysis of this selection process requires the list of the clutter region indices that intersect the region $\bigcup_{m=1}^M SI_m$. The list of indices is given by $L = \{c_{min}, \dots, c_i, \dots, c_{max}; O_c \cap \bigcup_{m=1}^M SI_m \neq \emptyset, c=1, \dots, C\}$. Also the analysis requires the volume of the intersection of the clutter region O_c and $\bigcup_{m=1}^M SI_m$. This is represented by a_c with the set of all volumes represented by $a = \{a_c\}_{c \in L}$. Note that these volumes are over the coordinates that have a uniform clutter distribution and SI_m includes all coordinates.

To simplify the number of events, target measurements for each model are only assumed to be within the I nearest. When not in the I nearest, they are assumed to be outside the remaining $IM - I$ measurements to all other models. This assumption requires the probability of the target being in $\overline{SI}_m \cap \bigcup_{n=1, n \neq m}^M SI_n$ to be much less than for it being in \overline{SI}_m .

The selection approach steps through each model and finds the I nearest measurements. These measurements are picked from those within a search region placed about the predicted position and which contains at least I measurements. This step is followed by finding the number of measurements in the list of IM that are in each clutter region. These are represented by the set $n = \{n_c\}_{c \in L}$.

The pseudo code for the selection procedure then simply reduces to the following steps.

For $m=1$ to M

From the data collected at time k place a search region about the predicted position $H\hat{x}_m(k | k-1)$ and evaluate the distance term, $d_{i,m}$, defined by equation (4).

Select the set of measurements $\{y_{i,m}\}_{i=1}^I$ with the I smallest $d_{i,m}$ values.

Establish pointers for common measurements and determine IM

¹ The symbols for the measurement space regions that are associated with the selection step are in italics. This is to help distinguish them from event space partitions that are non-italicized.

EndFor m

For $c = 1$ to C

Search $\{y_{i,m}\}_{i=1}^I$ to find the number of measurements in each clutter region, ie
 $\{n_c\}_{c \in L}$.

EndFor c

To simplify the volume calculations, the regions enclosing the I nearest measurements to each model m are represented by rectangular parallelepipeds. Figure 6 gives a two dimensional example for the case of 3 models selecting the 2 nearest measurements.

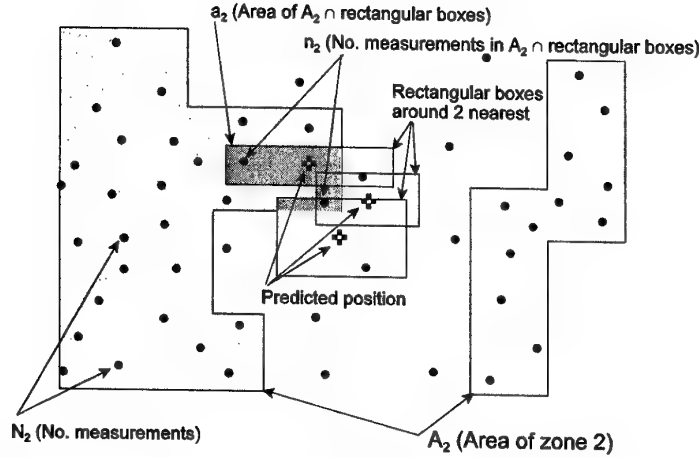


Figure 6. Example of selection boxes and clutter regions.

The filter derivation uses both the I nearest measurements and the list of IM unique measurements. To distinguish between the variables associated with these two lists, italics identifies the I nearest and non-italics the IM unique list. With this notation, all the I nearest measurements to each model are represented by $Y_m = \{y_{i,m}\}_{i=1}^I$ and the list of unique measurements by $Y = \{y_i\}_{i=1}^{IM}$. Similarly the clutter region index for measurement y_i is c_i while for measurement $y_{i,m}$ it is $c_{i,m}$.

3.5.1. Target and Clutter Selection Probabilities

The symbol Ps_m denotes the *a priori* probability that a target measurement is within the surveillance region given the target obeys the m^{th} model, is visible and is detected. This probability involves integrating Equation (5) for the m^{th} model over the surveillance region S and is given by,

$$Ps_m = \int_{y \in S} f_m(y) dy. \quad (8)$$

If the selection process includes signal level, then the integral over S in amplitude is from the detection threshold to infinity. Another factor that reduces Ps_m occurs with a step scanned sensor when a target approaches the region boundary for data gathered at each step position. S is then limited against the edge of the region boundary. When a track is on the edge, Ps_m is reduced by half.

The probability that a target measurement is one of the I nearest measurements to model m is denoted by PSI_m . This is defined given the target is represented by model m and involves integrating over the region SI_m . SI_m is normally symmetrical about the predicted position for model m except when it intersects S . Figure 7 illustrates the case when SI_m truncates the boundary of S . Therefore, the probability is given by:

$$PSI_m = \int_{y \in SI_m} f_m(y) dy. \quad (9)$$

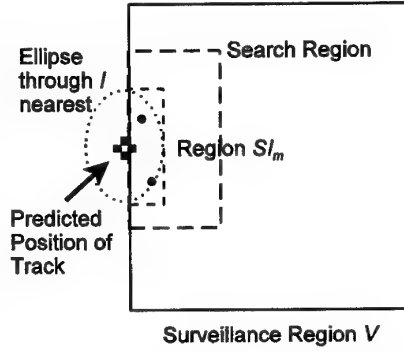


Figure 7. Example of region truncation.

The probability density for clutter measurements, $g_c(y)$, is defined within the clutter region O_c . The assumption is that $a_c < A_c$. That is, the clutter densities are largely based on measurements not selected by the track. This helps validate the *a priori* assumption for a clutter distribution that is computed from the measurements received at time k .

The probability that a clutter measurement is within the intersection of the clutter region O_c and the selection regions SI is given by:

$$Pc_c = \frac{a_c}{A_c} \int_{s \in O_c \cap \bigcup_{m=1}^M SI_m} g_c(s) ds. \quad (10)$$

PSI_m is computed over the region SI_m that contains the I nearest to model m and Pc_c uses both the region of the I nearest and the clutter regions O_c . Because SI_m is dependent on the data received at time k , an *a priori* density is needed to give the probability of getting the measured values a and n which are contained in the SI_m regions. Since the areas a are derived from SI and the clutter regions O the *a priori* probability is written as $p(n, SI | X_{i,m}, N, O, Z)$, where $X_{i,m}$ is the event that the i^{th} measurement is from a target with model m . The case of $i = 0$ denotes all measurements from clutter. To find an expression for this density, standard conditional probability expansions let it be expressed as the product of $p(n | X_{i,m}, SI, N, O, Z)$ and $p(SI | X_{i,m}, N, O, Z)$.

An expression for $p(n | X_{i,m}, SI, N, O, Z)$ is found by firstly assuming the clutter region parameters are independent of one another. This leads to the density being given by the produce over all $c \in L$ of $p(n_c | X_{i,m}, SI, N_c, O_c, Z)$. The probability of getting n_c clutter measurements, ie event $X_{0,m}$, is represented by the binomial distribution, namely

$$p(n_c | X_{0,m}, \dots) = \binom{N_c}{n_c} Pc_c^{n_c} (1 - Pc_c)^{N_c - n_c} \quad (11)$$

When one of the I nearest measurements is from a target, there are $n_c - 1$ clutter measurements selected from $N_c - 1$. The density is then given by,

$$p(n_c | X_{i,m} \dots) = \binom{N_c - 1}{n_c - 1} P_{c_c}^{n_c - 1} (1 - P_{c_c})^{N_c - n_c} \quad (12)$$

The simplest option to find an expression for $p(SI | X_{i,m}, N, O, Z)$ is to assume that this density is the same for all i and m . This term then cancels in the event probability solution which uses Bayes' rule to rearrange the conditional probabilities. This assumption is valid for large I . Tests on experimental data were undertaken to determine if there was a noticeable difference in the density $p(SI | X_{i,m}, N, O, Z)$ for $i = 0$ and $i > 0$ when $I < 5$. These test showed no consistent difference. Therefore the assumption is made that the *a priori* density for selection region size is the same for all i and m .

4. Sample Space for Sensor Measurements

From the Sensor and Environment Model presented in Section 3, the measurements selected for each track originate either from the target associated with the track or from clutter. The case of measurements from other targets is not considered in this analysis. Each process discussed in Section 3 corresponded to a process in the Sensor and Environment Model. These processes take the data from a previous step and transforms it to a new data set. This action can be represented as a mapping from an input sample space to an output sample space[8,10]. The output sample space for a process output contains all possible combinations of data from that process. The action of the process, eg formation of measurements, partitions the output sample space. Figure 8 shows the possible paths that target returns can take through the sensor and environment model in Figure 3. Figure 9 gives the associated sample space for the selected measurements with the partitions formed by the paths in Figure 8.

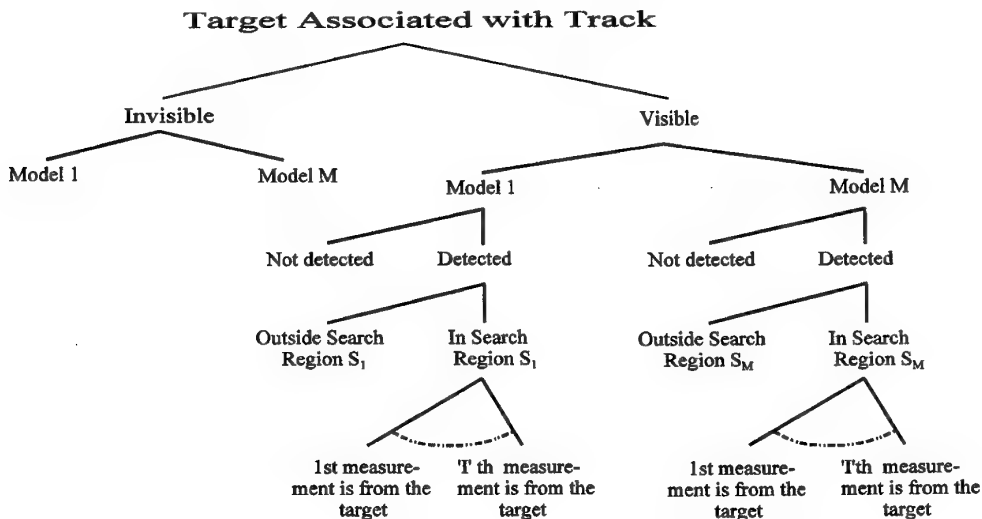


Figure 8. Path of Target and Sensor Returns

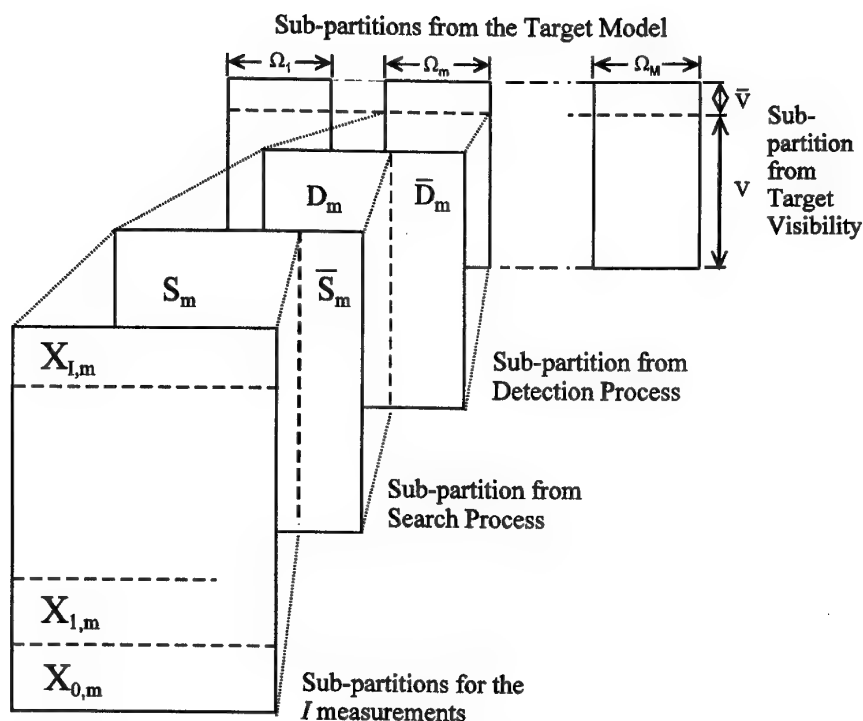


Figure 9. Sample Space for Selected Measurements

In Figure 9 both the target visibility (V and \bar{V}) and the M target models ($\Omega_m, m=1, \dots, M$) partition the target sample space. This is because these events are part of the target, environment and propagation models and processes.

The next process is the measurement process which partitions each of the $V \cap \Omega_m, m=1, \dots, M$. If the target associated with the track is visible and obeys the m^{th} model, it is either detected, D_m , or undetected, \bar{D}_m .

The selection process then partitions $D_m, m=1, \dots, M$. The surveillance region creates the first partitions S_m and \bar{S}_m of D_m . Each S_m is further partitioned by the selection of the nearest I measurements to each model and the removal of duplicate measurements to form the measurement set Y . The assumption that target measurements are not in the $IM - I$ measurements that are outside the I nearest leads to the $X_{i,m}$ events where the i^{th} nearest measurement is from the m^{th} target model, $i=0, \dots, I$ and $m=1, \dots, M$. The case $i=0$ denotes the event when the target measurement from the m^{th} model is not contained in the IM measurements.

5. Target State Estimate

From section 3, the data provided by the sensor with each multiple model track consists of the selected measurements $Y(k)$, the regions containing the I nearest measurement SI , the clutter region parameters at time k , ie n, N, O and the past track data $Z(k-1)$. For notational clarity the time dependence of the sensor data is not shown. From the defined target and sensor model in Section 3 and the partitions defined in Section 4, the optimal estimate of the target state $x(k)$ is,

$$\begin{aligned}\hat{x}(k|k) &= E\{x(k)|Y, SI, n, N, O, Z\} \\ &= \sum_{m=1}^M \sum_{i=-1}^I \beta_{i,m} E\{x_m(k)|\Theta_{i,m}, Y, SI, n, N, O, Z\}.\end{aligned}\quad (13)$$

Where

$$\begin{aligned}\beta_{i,m} &= \Pr\{\Theta_{i,m}|Y, SI, n, N, O, Z\} \\ \Theta_{i,m} &= \{\text{Measurement } y_i \text{ is from a target represented by the } m^{\text{th}} \text{ model}\} \\ &= V \cap \Omega_m \cap D_m \cap S_m \cap X_{i,m} \quad i = 1, \dots, I \quad m = 1, \dots, M \\ \Theta_{0,m} &= \{\text{All } IM \text{ measurements are from clutter and the target is Visible and represented by the } m^{\text{th}} \text{ model}\} \\ &= V \cap \Omega_m \cap (\overline{D}_m \cup D_m \cap (\overline{S}_m \cup S_m \cap X_{0,m})) \quad m = 1, \dots, M \\ \Theta_{-1,m} &= \{\text{All } I \text{ measurements are from clutter and the target is Not Visible and represented by the } m^{\text{th}} \text{ model}\} \\ &= \overline{V} \cap \Omega_m\end{aligned}$$

To avoid an extra summation for the visible and invisible cases, the index range of i is extended to -1 to denote the invisible event. In this way the summation indexes through all target models for the event of an invisible target. This does not occur when an additional model index is added for the event of an invisible target.

The assumption that a target measurement is within the I nearest simplifies the event $\Theta_{0,m}$ by not including the case of the target measurement within S_m but outside the IM measurements. The evaluation of the probability related to this term involves integrating over intersecting parallelepipeds - a complicated procedure to evaluate terms that have a very low probability.

The evaluation of $E\{x_m(k)|\Theta_{i,m}, Y, SI, n, N, O, Z\}$ for $i > 0$ is the Kalman filter solution for updating the predicted state, $\hat{x}_m(k|k-1)$, of a target represented by model m , with measurement y_i . Thus, the evaluation of equation (13) is given by the following.

$$\hat{x}(k|k) = \sum_{m=1}^M \sum_{i=-1}^I \beta_{i,m} \hat{x}_{i,m}(k|k) \quad (14)$$

$$\begin{aligned} \text{Where } \hat{x}_{i,m}(k|k) &= \hat{x}_m(k|k-1) + \mathbf{W}_m \tilde{y}_{i,m} & i = 1, \dots, I \\ \hat{x}_{-1,m}(k|k) &= \hat{x}_m(k|k-1) & i = -1, 0 \\ \mathbf{W}_m &= \left(\mathbf{F}_m \mathbf{P}_m(k-1|k-1) \mathbf{F}_m^T + \mathbf{Q}_m \right) \mathbf{H}^T \mathbf{S}_m^{-1}, \text{ is the gain matrix for} \\ &\text{the } m^{\text{th}} \text{ model.} \end{aligned}$$

From equation (14), each model is similar to a PDA filter. These remain separate with the contribution of a model to the state estimate tending to zero as its probability approaches zero. This filter therefore combines the approaches taken in PDA and track split.

The selection process requires the estimate of each model's state given it is valid. This is obtained by writing equation (13) in the following form.

$$\hat{x}(k|k) = \sum_{m=1}^M \beta_m \hat{x}_m(k|k). \quad (15)$$

$$\begin{aligned} \text{Where } \beta_m &= \Pr\{\Omega_m | Y, SI, n, N, O, Z\} = \sum_{i=-1}^I \beta_{i,m} \\ \hat{x}_m(k|k) &= E\{x_m(k) | \Omega_m, Y, SI, n, N, O, Z\}. \end{aligned}$$

From equations (14) and (15), the estimate of each model's state is then given by,

$$\hat{x}_m(k|k) = \hat{x}_m(k|k-1) + \mathbf{W}_m \tilde{y}_m(k). \quad (16)$$

$$\text{Where } \tilde{y}_m = \frac{\sum_{i=1}^I \beta_{i,m} \tilde{y}_{i,m}}{\beta_m}$$

Now the covariance of $\hat{x}(k|k)$ is a weighted sum of Gaussians where each is derived from the PDAF approximation for each model. The event probabilities for each model require the covariance for $\hat{x}_m(k|k)$ which is denoted by $\mathbf{P}_m(k|k)$. This is derived in Appendix A and is given by the expression.

$$\begin{aligned} \mathbf{P}_m(k|k) &= \frac{\beta_{-1,m}}{\beta_m} \mathbf{P}_{-1,m}(k|k-1) + \frac{\beta_{0,m}}{\beta_m} \mathbf{P}_m(k|k-1) + \left(1 - \frac{\beta_{-1,m} + \beta_{0,m}}{\beta_m} \right) \mathbf{P}^m(k|k) \\ &\quad + \mathbf{W}_m \left(\sum_{i=1}^I \frac{\beta_{i,m}}{\beta_m} \tilde{y}_{i,m} \tilde{y}_{i,m}^T - \tilde{y}_m \tilde{y}_m^T \right) \mathbf{W}_m^T \end{aligned} \quad (17)$$

Where $\mathbf{P}^m(k|j)$ is the Kalman filter covariance for model m at time k given data to time j .

$P_{-1,m}(k|j)$ is the covariance for model m when the target is invisible at time k given data to time j .

As the filter operations only require the state estimate $\hat{x}(k|k)$ to display track position, the associated covariance is not required and is therefore not calculated.

The derivation of the event probabilities $\beta_{i,m}$ required for equations (14) and (17) is given in Appendix B. These probabilities simplify to,

$$\beta_{i,m} = \frac{b_{i,m}}{\sum_{l=1}^M \sum_{j=-1}^I b_{j,l}}. \quad (18)$$

Where

$$\begin{aligned} b_{i,m} &= Pm_m Pd_m Ps_m \frac{f_m(y_{i,m}) n_{c_{i,m}} A_{c_{i,m}}}{g_{c_{i,m}}(s_{i,m}) N_{c_{i,m}} I} & 1 \leq i \leq I, \quad 1 \leq m \leq M \\ b_{0,m} &= Pm_m \left(1 - Pd_m Ps_m + Pd_m (Ps_m - Psl_m) \sum_{c \in L} \frac{N_c - n_c}{N_c (1 - Pc_c)} \right) & 1 \leq m \leq M \\ b_{-1,m} &= Pm_m \frac{(1 - Pv)}{Pv} & 1 \leq m \leq M \end{aligned}$$

Because the target is assumed not to change between models (see Section 3.2), the *a priori* model probability Pm_m is the model probability from the previous scan, ie $Pm_m = \beta_m(k-1|k-1)$.

The *a priori* visibility Pv is given by equation (2). From the event probabilities $\beta_{i,m}$ the *a posteriori* target visibility is given by,

$$Pv(k|k) = 1 - \sum_{m=1}^M \beta_{-1,m} \quad (19)$$

This visibility is then used in equation (2) as the past data in the next time step with the *a priori* transition probabilities.

The target visibility defined by equation (19), forms the basis for decisions relating to the change from the tentative state to the pending state. In practice the rate of change in target visibility needs to be reduced by a low pass filter before using it for state transition decisions. Once a track is in the pending state, its transition to the established state is determined by one model becoming dominant. This is found from the probability $1 - \beta_{-1,m}$.

6. Implementation and Performance

The multiple model initiation filter has been implemented and installed on the Jindalee Over-the-Horizon Radar (OTHR) at Alice Springs. This radar detects targets by sequentially stepping through regions within the surveillance area. At each step position the receiving system coherently integrates the backscattered radar returns. The coherent processing gives the signal power in the dimensions of range, azimuth and Doppler (ARD). This data are then whitened[16] and peak detected[12] to locate possible target returns in the ARD dimensions. Each measurement from the peak detector has the interpolated range, azimuth, Doppler and SNR as well as the clutter region index c . The peak detector also calculates the number of peaks N_c within each clutter region. In this implementation, the peak detections from clutter are assumed to be uniform within a clutter region over the ARD dimensions and non-uniform in SNR. The whitening process therefore gives the area A_c of each clutter region in the ARD dimensions. This data is passed to the tracking system. Prior to any tracking, a separate processing step updates the clutter region SNR histograms and derives the parameters for the SNR probability density function[15].

The tracking system uses the above data to perform track update and initiation as described in the following section. This is followed by examples of tracking performance on recorded OTHR data.

6.1. Overview of Filter Implementation

The main operations performed by the tracking system are shown below in Figure 10. This figure uses the track states defined in section 2, page 4. The tracking system tracks are contained in separate global arrays for Established, Pending and Tentative tracks. On receipt of new peak detector data, the tracks in the area covered by this data are copied from these global arrays to working arrays. The first process involves the update of Established tracks by the Established Tracker. The filter used here is a single model filter that has been altered to incorporate the clutter regions in the same way as for the multiple model filter. The final Established Tracker step is the cancellation of the peaks most likely to have come from targets. The following process uses the uncanceled peaks for the update of the Pending tracks by the Pending Multiple Model Tracker. The final step in this tracker is to cancel likely target peaks. These uncanceled peaks are then used for the Tentative Multiple Model Tracker which includes in its final step the cancellation of peaks.

New tracks enter the system after these actions by choosing likely target peaks from the uncanceled peak detections. The selected peaks provide the initial state parameters for range, azimuth and SNR. The ambiguity velocity combined with peak Doppler gives the radial velocity for each of the filter models. The covariance values for these initial values are derived from preset values for the measurement noise covariance.

When all tracks have been updated and initiated, the final step promotes these new and working tracks to the global track arrays. The new tracks become Tentative tracks for the next update. The promotion of tracks from Tentative to Pending and from Pending to Established is governed by the track visibility and model probability.

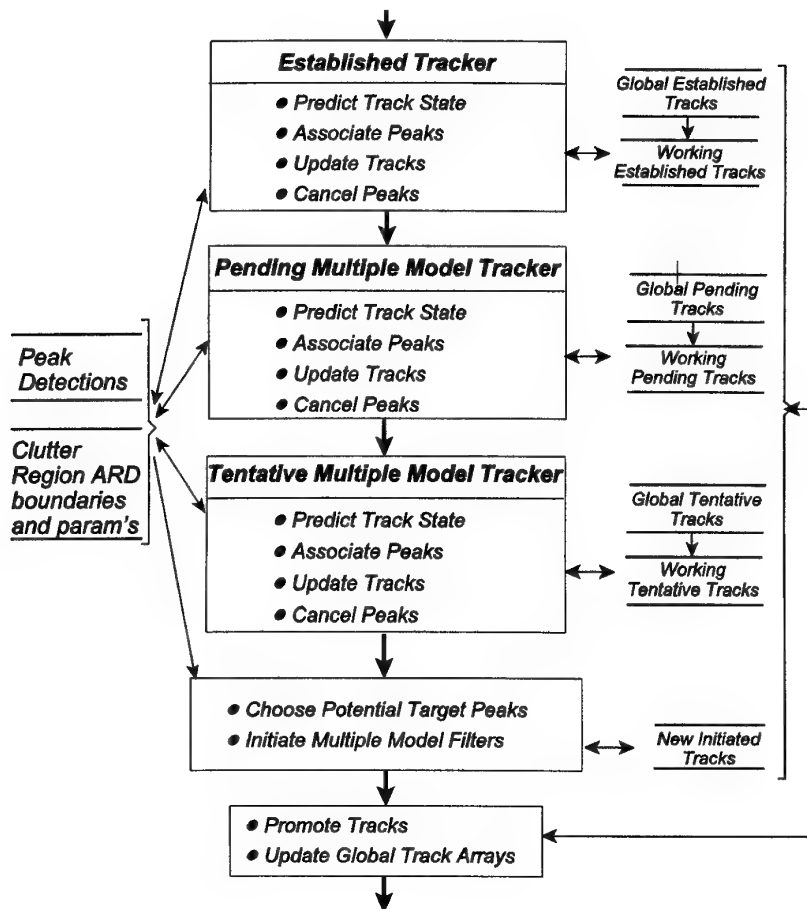


Figure 10. An OTHR tracking system using a multiple model initiation filter.

The way in which SNR is tracked by this filter differs to that proposed by Lerro et al [4]. He modelled the target SNR by a Raleigh distribution. An estimate of the expected SNR is derived from a fixed gain recursive filter of the measured values at each update. The filter is initialized by a preset minimum SNR value. This approach suffers from the following deficiencies:

- A fixed gain filter is less optimal than a PDA filter if estimating the mean SNR.
- Initiation of the SNR by a preset minimum value is only correct for one value, all other mean SNR values will take longer for the correct estimate to be formed.

The importance of correctly modelling the SNR is critical because it is incorporated into the event probability formulae via a likelihood ratio (see equation (18)). Errors in the clutter measurement pdf will either lead to optimistic or pessimistic event probabilities. Optimistic probabilities give increased track divergence while pessimistic values give

sub-optimal performance. The above deficiencies give sub-optimal performance at the most critical time of tracking.

The SNR tracking approach used here gives a more optimal approach and follows that given in [9,10,11]. That is, SNR is treated the same as the other tracking coordinates. To do this the SNR target measurement scatter is modelled by a normal distribution about a mean value. In this application the mean value is an unknown constant value with no SNR rate of change term and a modest plant noise to cater for SNR change with target range. To achieve an approximately normal distribution for the scatter of SNR measurements, the processed ARD data has the log function applied to the SNR values. From analysis of the SNR of target measurements in recorded OTHR data, a heuristic model was derived for the SNR measurement noise dependence on the mean value. Hence the SNR filter is initialized with the SNR of the peak that initiates the track in the other coordinates and the measurement variance is set from the heuristic model. This filter is then updated in the same manner as the filters for the other ARD coordinates. In this way the SNR estimate converges in the same way as the estimates for the other tracking coordinates.

6.2. Performance Examples

The examples of tracking performance for the tracking filter implementation described in the previous section uses recorded data from the Jindalee OTHR. The examples compare the new advanced tracker with the original Jindalee tracker[9]. Because the advanced tracker uses a new peak detector and whitener, this comparison includes both of the tracking filters with their required whitening and detection steps. A summary of the detection and tracking algorithm differences are shown in the table below.

Table 1. Summary of the detection and tracking algorithm differences in comparison.

PROCESSING STEPS	DETECTION AND TRACKING SYSTEMS	
	ORIGINAL TRACKER	ADVANCED TRACKER
Whitener	Trimmed mean over bands in Doppler and range.	Greatest of Order Statistic whitener
Peak Detector	Detection and interpolation on whitened data.	Peak detection and interpolation on pre-whitened data with clutter region index added to peaks. ARD area of clutter regions passed to tracker.
Track Initiation	Includes target Visibility and estimates SNR. Clutter model has a non-uniform pdf in SNR and uniform over predefined Doppler bands.	Multiple Model Initiation filter as described in this paper with clutter regions and adaptive non-uniform model in SNR.
Established Tracking	Same as for initiation but with different target manoeuvre and Visibility parameters.	Revised original tracker with clutter regions added as for Multiple Model filter.

Note that the inclusion of the different whitener, peak detector and established tracking filter help contribute to the performance improvements from the advanced tracking system. The reader is reminded that the new filter must have the modified peak detector because of its requirement for clutter region data. Furthermore the established tracking filter must be able to use the output from the new peak detector.

The advanced tracking system improves the initiation on transponder returns and reduces the number and length of false tracks compared with the original tracker. The first example in Figure 11 shows the improvement in initiation of tracks on transponder returns. The range versus time display on the left is the output from the original tracker. Stationary tracks are displayed as black crosses while moving tracks are grey crosses. When a track is in the pending state, it can change between stationary and moving. This is the reason why some of the tracks in the right range-time display in Figure 11 change between black and grey crosses at the start. The cross position is the filter's state estimate when the filtered visibility is above the display threshold. A line joins the crosses when the filtered visibility is below the display threshold. The numbered labels to identify tracks in the following description are placed as close as possible to the start of the track.

The OTHR track data in Figure 11 was recorded under conditions of severe multi-mode propagation. The system forms a track on each mode that is detected and in the range versus time format, the multi-mode tracks are displayed as parallel tracks.

The first notable feature of the advanced tracker in Figure 11 is the reduced initiation time. Here tracks appear in under one third the time of the original tracker. The transponder is turned on just before the second time mark on the horizontal axis. The advanced tracker output on the right clearly shows the multi-mode tracks on the transponder returns. The transponder tracks are labelled '1'. The stationary tracks labelled '2' are produced by an incorrectly initiated stationary track passing over the multi-mode target returns. The stationary track along the bottom of the range-time display, labelled '3', is from a locally injected calibrate signal.

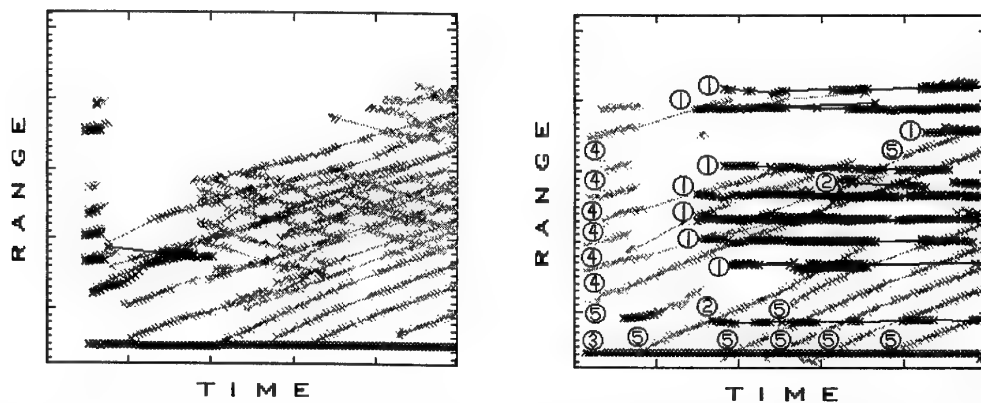


Figure 11. Original and advanced tracker performance when transponder turned on.

There are two outbound targets. The first is labelled '4' and lands at the location of the transponder. The second is labelled '5' and is yet to land. Both of these targets have at least 6 multi-mode tracks. The other outbound multi-mode tracks '5' are tracked similarly by the two trackers.

The original tracker output on the left of Figure 11 produces a number of moving tracks when the transponder is turned on. This happens because the stationary option is considered only for about 5 minutes after the start of a radar mission. This approach was used to remove the incidence of falsely initiating a slow moving target as a stationary track (see page 1). In the original design, it was assumed that all stationary targets such as transponders and calibrate signals would be present at the start. In practice, ionospheric propagation modes change with time as evident in the track output near the end of the time axis in Figure 11. Because the original tracker promotes both the stationary and the most confident moving track, these are both present on the target that lands, ie '4'.

The second example in Figure 12 demonstrates the reduction in false tracks using night-time data. Spread clutter was present in this data and generated several moving tracks in the original tracker. The advanced tracker output on the right has significantly fewer and shorter duration false tracks. The valid moving tracks in this data are labelled '1' and the tracks from a calibrate signal are labelled '2'. This data has two calibrate signals

and the original tracker did not successfully initiate a stationary track on one of these. This lead to the repetitive formation of moving tracks.

Once again the advanced tracker has better than a third the initiation time delay. Thus with this tracker it has been possible to counter the normal approach that is taken to reduce false tracks, namely to increase the initiation delay. The reason for the reduction in false tracks is the clutter map information that is incorporated into the tracker.

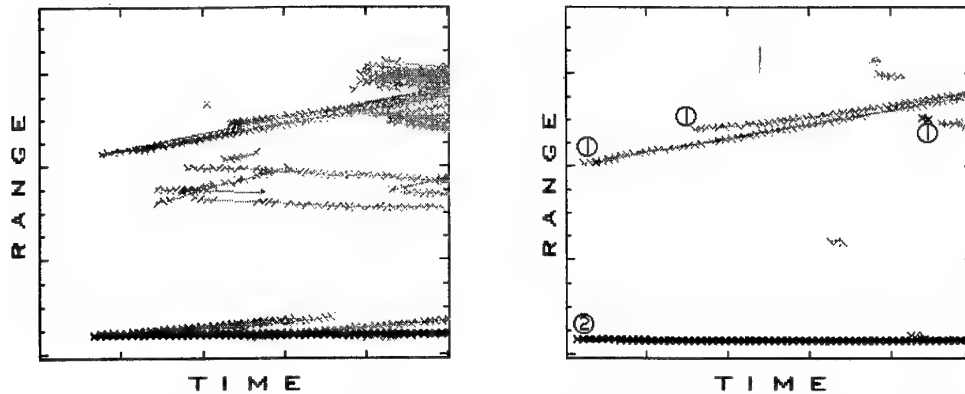


Figure 12. Original and advanced tracker performance with cluttered night-time data.

7. Conclusions

The tracking filter outline in this paper minimises the estimation errors from when a filter is initiated to when it is presented for display because it has parallel PDA filters that span the target's parameter range. The update of these filters includes an extra event for the target model being correct. This alters the event probability for measurements shared by more than one filter model and so reduces the time to resolve the target's model. Hence the time to display a track is reduced and the likelihood of getting the correct model is enhanced.

The filter also includes a clutter and noise model for the measurements supplied to the tracker to allow for them to be both non-uniform and from different clutter regions. The filter update algorithm then incorporates the non-uniformity and clutter regions of the measurements. It also maintains a constant number of measurements to keep a fixed number of events. All these changes have demonstrated significant reduction in the number of false tracks.

Quantitative assessment of the improvement in performance from this new filter will be given in a separate paper.

8. Acknowledgements

The validation of the filter formulation, that is described in this paper, involved frequent comparison between theory and practice. I wish to thank Samuel Davey for his splendid effort in implementing the filter on the Jindalee Replay system. He also provided both valuable insights into the actions of the filter and suggestions on the formulation. In addition, I wish to thank Branko Ristic for his formulation and implementation of the adaptive clutter density model. His assistance with this work and the checking of the filter formulation helped to bring this paper up to its current standard. Thanks to Samuel and Branko!

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Appendix A: Derivation of the Filter Covariance

Here the derivation of the covariance for the estimate of each model is derived. Similar procedures can be used for the target state estimate. However, this is not required for any of the filter operations.

The covariance of the state estimate $\hat{x}_m(k|k)$ involves evaluating the following expression.

$$\mathbf{P}_m(k|k) = \int (x(k) - \hat{x}_m(k|k))(x(k) - \hat{x}_m(k|k))^T p_m(x(k)) dx(k) \quad (\text{A-1})$$

From equation (15), the probability density $p_m(x)$ is given by

$$p_m(x) = \sum_{i=-1}^I \frac{\beta_{i,m}}{\beta_m} p(x_m | \Theta_{i,m}, Y, SI, n, N, O, Z). \quad (\text{A-2})$$

Where $p(x_m | \Theta_{i,m}, Y, SI, n, N, O, Z)$ is the pdf of $x_m(k)$ given event $\Theta_{i,m}$ and all data. Equation (A-1) can then be written as,

$$\mathbf{P}_m(k|k) = \sum_{i=-1}^I \frac{\beta_{i,m}}{\beta_m} \text{Integral}_{i,m}. \quad (\text{A-3})$$

Where $\text{Integral}_{i,m} = \int (x - \hat{x}_m)(x - \hat{x}_m)^T p(x | \Theta_{i,m}, Y, n, a, N, A, Z) dx$.

By substituting $[(x - \hat{x}_{i,m}) + (\hat{x}_{i,m} - \hat{x}_m)]$ for $(x - \hat{x}_m)$, $\text{Integral}_{i,m}$ becomes,

$$\begin{aligned} \text{Integral}_{i,m} = & \int (x - \hat{x}_{i,m})(x - \hat{x}_{i,m})^T p(x | \Theta_{i,m}, Y, SI, n, N, O, Z) dx \\ & + \int (x - \hat{x}_{i,m}) p(x | \Theta_{i,m}, Y, SI, n, N, O, Z) dx (\hat{x}_{i,m} - \hat{x}_m)^T \\ & + (\hat{x}_{i,m} - \hat{x}_m) \int (x - \hat{x}_{i,m})^T p(x | \Theta_{i,m}, Y, SI, n, N, O, Z) dx \\ & + (\hat{x}_{i,m} - \hat{x}_m)(\hat{x}_{i,m} - \hat{x}_m)^T \int p(x | \Theta_{i,m}, Y, SI, n, N, O, Z) dx. \end{aligned} \quad (\text{A-4})$$

Expansion of the 2nd and 3rd terms in equation (A-4) includes the integral for the mean given event $\Theta_{i,m}$ which, from the PDA assumption that the past data are represented by a single mean for each model, is $\hat{x}_{i,m}$. So these terms are zero. The 4th term integrates

to 1 while the first term is the covariance when the target satisfies model m and is represented by the term $\mathbf{P}^m(k|k)$. Thus, equation (A-3) simplifies to,

$$\mathbf{P}_m(k|k) = \sum_{i=1}^I \frac{\beta_{i,m}}{\beta_m} \mathbf{P}^m(k|k) + \sum_{i=1}^I \frac{\beta_{i,m}}{\beta_m} \hat{\mathbf{x}}_{i,m} \hat{\mathbf{x}}_{i,m}^T - \hat{\mathbf{x}}_m \hat{\mathbf{x}}_m^T \quad (\text{A-5})$$

Further simplification is achieved by expanding $\hat{\mathbf{x}}_{i,m}$ and $\hat{\mathbf{x}}_m$ and the covariance reduces to,

$$\begin{aligned} \mathbf{P}_m(k|k) = & \frac{\beta_{-1,m}}{\beta_m} \mathbf{P}_{-1,m}(k|k-1) + \frac{\beta_{0,m}}{\beta_m} \mathbf{P}_m(k|k-1) + \sum_{i=1}^I \frac{\beta_{i,m}}{\beta_m} \mathbf{P}^m(k|k) \\ & + \mathbf{W}_m \left(\sum_{i=1}^I \frac{\beta_{i,m}}{\beta_m} \tilde{\mathbf{y}}_{i,m} \tilde{\mathbf{y}}_{i,m}^T - \tilde{\mathbf{y}}_m \tilde{\mathbf{y}}_m^T \right) \mathbf{W}_m^T \end{aligned} \quad (\text{A-6})$$

Where $\mathbf{P}^m(k|j)$ is the Kalman filter covariance for model m at time k given data to time j .

$\mathbf{P}_{-1,m}(k|j)$ is the covariance for model m when the target is invisible at time k given data to time j .

Appendix B: Event Probability Derivation

The evaluation of the event probabilities $\beta_{i,m}$ for equation (13) follows the standard approach which uses Bayes' Rule to give.

$$\begin{aligned}\beta_{i,m} &= \Pr\{\Theta_{i,m} | Y, SI, n, N, O, Z\} \\ &= \frac{p(\Theta_{i,m}, Y, SI, n, N, O | Z)}{\sum_{l=1}^M \sum_{j=-1}^I p(\Theta_{j,l}, Y, SI, n, N, O | Z)}\end{aligned}\quad (B-1)$$

The density in the numerator of equation (B-1) is evaluated for the three separate cases of (1) $1 \leq i \leq I, m \geq 1$; (2) $i = 0, m \geq 1$; and (3) $i = -1, m \geq 1$. Note that the upper range limit of i is I because of the assumption that if the target measurement is not within the I nearest it is not within the remaining $IM - I$.

(1) Target is Visible and Measurement i is from Target, ie. $1 \leq i \leq I, m \geq 1$

The derivation starts with the definition of $\Theta_{i,m}$ in equation (13), substituted into the numerator of equation (B-1). The terms on the left-hand side of the conditional probability are rearranged and then expanded into the following product of conditional probabilities.

$$\begin{aligned}p(Y, n, X_{i,m}, SI, N, O, S_m, D_m, \Omega_m, V | Z) &= \\ & p(Y | n, X_{i,m}, SI, N, O, S_m, D_m, \Omega_m, V, Z) \\ & p(n | X_{i,m}, SI, N, O, S_m, D_m, \Omega_m, V, Z) \\ & p(X_{i,m} | SI, N, O, S_m, D_m, \Omega_m, V, Z) \\ & p(SI | N, O, S_m, D_m, \Omega_m, V, Z) \\ & p(N, O | S_m, D_m, \Omega_m, V, Z) \\ & p(S_m | D_m, \Omega_m, V, Z) \\ & p(D_m | \Omega_m, V, Z) p(\Omega_m | V, Z) p(V | Z)\end{aligned}\quad (B-2)$$

Each term is now taken in turn to find the solution.

The first term on the R.H.S. is the conditional joint density with the target and clutter measurements given they satisfy the event $X_{i,m}$. Hence,

$$p(Y|n, SI, X_{i,m}, N, O, S_m, D_m, \Omega_m, V, Z) = \frac{f'_m(y_{i,m})}{g'_{c_{i,m}}(y_{i,m})} \prod_{j=1}^{IM} g'_{c_j}(y_j) \quad (B-3)$$

where

$g'_{c_{i,m}}(y_{i,m})$ is the probability density function for measurement $y_{i,m}$ being

from clutter given it is within $O_{c_{i,m}} \cap \bigcup_{m=1}^M SI_m$, ie $\frac{g_{c_{i,m}}(y_{i,m})}{Pc_{c_{i,m}}}$,

$f'_m(y_{i,m})$ is the probability density function for a target measurement given it is visible, obeying the m^{th} model, detected and within the region containing the I nearest measurements SI_m , ie $\frac{f_m(y_{i,m})}{PSI_m}$.

The second term in equation (B-2) is the joint density of the number of clutter measurements in SI as defined by the event $X_{i,m}$ and the clutter parameters N, O . The solution is given by equations (11) to (12) and is the product over all clutter regions, $c \in L$, of the Binomial distribution for getting $n_c - 1$ clutter measurements (see equation (12), page 15). Note that n_c is reduced by one for the clutter region c_i that contains the target measurement. To identify the variables that are not subsequently cancelled, the Binomial expression is given with all n_c being from clutter. Scaling terms multiply this expression to give the correct solution for the event $X_{i,m}$. So the second term is:

$$p(n|X_{i,m}, SI, N, O, S_m, D_m, \Omega_m, V, Z) = \frac{n_{c_{i,m}}}{N_{c_{i,m}} Pc_{c_{i,m}}} \prod_{c \in L} \binom{N_c}{n_c} Pc_c^{n_c} (1 - Pc_c)^{N_c - n_c} \quad (B-4)$$

The third term is the conditional probability of the event $X_{i,m}$. Since the range of i for the terms on the RHS of the conditional is 0 to I , the third term for $i = 1, \dots, I$ is PSI_m/I for the I possible target measurements.

The fourth term is the *a priori* probability $p(SI|N, O, \dots, Z)$, which involves the evaluation of the density for the expected selection regions based on the clutter and target related parameters. This density is assumed to be the same for all i and m . Therefore it is not included as it cancels in the solution.

The fifth term is the joint density of N, O which is also assumed to be the same for all models and events and therefore cancels. For simplicity, no symbol is used to denote this density.

The remaining terms $p(S_m | D_m, \Omega_m, V, Z)p(D_m | \Omega_m, V, Z)p(\Omega_m | V, Z)p(V | Z)$ are given by the product $Ps_m Pd_m Pm_m Pv$.

The combination of the above expressions gives the joint density as:

$$p(V, \Omega_m, D_m, S_m, X_{i,m}, Y, SI, n, N, O | Z) = Pv Pm_m Pd_m Ps_m \frac{P_{SI} f'_m(y_{i,m}) n_{c_{i,m}}}{g'_{c_{i,m}}(y_{i,m}) N_{c_{i,m}} P_{c_{i,m}} I} \prod_{j=1}^{IM} g'_{c_j}(y_j) \prod_{c \in L} \binom{N_c}{n_c} P_{c_c}^{n_c} (1 - P_{c_c})^{N_c - n_c} \quad (B-5)$$

(2) **Target is Visible and No Measurements are from model m , ie. $i = 0, m \geq 1$**

The case of the target measurement being in the remaining IM measurements is not considered. Therefore the ways in which none of the selected measurements are from a target represented by model m when the target is visible is as given in the definition of $\Theta_{0,m}$ on page 18 and can be written as,

$$p(\Theta_{0,m}, Y, n, a, N, O | Z) = p(V, \Omega_m, \bar{D}_m, Y, n, SI, N, O | Z) + p(V, \Omega_m, D_m, \bar{S}_m, Y, n, SI, N, O | Z) + p(V, \Omega_m, D_m, S_m, X_{0,m}, Y, n, SI, N, O | Z). \quad (B-6)$$

The previous approach for expanding the conditional probabilities is used for each component to find the probability density function.

The first term is the probability density function for a visible m^{th} model which is not detected. Therefore, all the selected measurements are from clutter. From the procedures used to give equation (B-5) this density is given by,

$$p(V, \Omega_m, \bar{D}_m, Y, n, SI, N, O | Z) = Pv Pm_m (1 - Pd_m) \prod_{j=1}^{IM} g'_{c_j}(y_j) \prod_{c \in L} \binom{N_c}{n_c} P_{c_c}^{n_c} (1 - P_{c_c})^{N_c - n_c} \quad (B-7)$$

The second density in equation (B-6) is for a visible m^{th} model with the target detected but its measurement lies outside the surveillance region. As above all the selected measurements are from clutter. The density is then,

$$p(V, \Omega_m, D_m, \bar{S}_m, Y, n, SI, N, O | Z) = Pv Pm_m Pd_m (1 - Ps_m) \prod_{j=1}^{IM} g'_{c_j}(y_j) \prod_{c \in L} \binom{N_c}{n_c} P_{c_c}^{n_c} (1 - P_{c_c})^{N_c - n_c} \quad (B-8)$$

The third term is for a visible m^{th} model with the target detected and within the surveillance region but not in one of the IM selected measurements. This joint density is given by,

$$\begin{aligned}
 & p(\bar{V}, \Omega_m, D_m, S_m, X_{0,m}, Y, n, SI, N, O | Z) \\
 &= P_v P m_m P d_m P s_m \left(1 - \frac{P_{SI_m}}{P s_m} \right) \\
 & \left(\prod_{j=1}^{IM} g'_{c_j}(y_j) \prod_{c \in L} \binom{N_c}{n_c} P c_c^{n_c} (1 - P c_c)^{N_c - n_c} \right) \sum_{c \in L} \frac{N_c + n_c}{N_c (1 - P c_c)}.
 \end{aligned} \tag{B-9}$$

This expression only considers the case of the target measurement being in the clutter regions for the I nearest measurements and not for all regions $c \in L$.

(3) Target is Invisible, ie. $i = -1, m \geq 1$

For an invisible target there are IM clutter measurements in the region $\bigcup_{m=1}^M SI_m$ with the number in each clutter region $c \in L$ given by n_c . The density is then given by,

$$\begin{aligned}
 & p(\bar{V}, \Omega_m, Y, n, SI, N, O | Z) = (1 - P_v) P m_m \\
 & \prod_{j=1}^{IM} g'_{c_i}(y_i) \prod_{c \in L} \binom{N_c}{n_c} P c_c^{n_c} (1 - P c_c)^{N_c - n_c}
 \end{aligned} \tag{B-10}$$

The solution to equation (B-1) now follows by substituting equations (B-5) to (B-10) into equation (B-1). To simplify the solution, terms that are identical for all i and m are cancelled, namely $\prod_{j=1}^{IM} g'_{c_j}(y_j) \prod_{c \in L} \binom{N_c}{n_c} P c_c^{n_c} (1 - P c_c)^{N_c - n_c}$. The solution can then be written in the following form.

$$\beta_{i,m} = \frac{b_{i,m}}{\sum_{l=1}^M \sum_{j=-1}^I b_{j,l}}. \tag{B-11}$$

Where

$$b_{i,m} = Pm_m Pd_m Ps_m \frac{f_m(y_{i,m}) n_{c_{i,m}} A_{c_{i,m}}}{g_{c_{i,m}}(s_{i,m}) N_{c_{i,m}} I} \quad 1 \leq i \leq I, \quad 1 \leq m \leq M$$

$$b_{0,m} = Pm_m \left(1 - Pd_m Ps_m + Pd_m (Ps_m - Ps_{l_m}) \sum_{c \in L} \frac{N_c - n_c}{N_c (1 - Pc_c)} \right) \quad 1 \leq m \leq M$$

$$b_{-1,m} = Pm_m \frac{(1 - Pv)}{Pv} \quad 1 \leq m \leq M$$

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19. ABSTRACT This paper presents the theory and examples of performance for a new algorithm that initiates tracks using a multiple model Probabilistic Data Association (PDA) filter. The analysis is generalised for the case of multiple non-uniform clutter regions within the measurement data that updates the filter. The algorithm starts multiple parallel PDA filters from a single sensor measurement. Each filter is assigned one of a range of possible target model parameters. To reduce the possibility of clutter measurements forming established tracks, the solution includes a model for a visible target. That is, a target that gives sensor measurements that satisfy one of the target models. Other features included in the algorithm are the selection of a fixed number of nearest measurements and the addition of signal amplitude to the target state vector. The inclusion of signal amplitude is one coordinate that is applicable to the non-uniform clutter model developed in this paper.					